Challenging Trigonometry Questions:

6. (a) For how many integers k with $0 < k < 18$ is $\dfrac{5\sin(10k^\circ) - 2}{\sin^2(10k^\circ)} \geq 2$?

possible?

(b) If $\cos\theta = \tan\theta$, determine all possible values of $\sin\theta$, giving your answer(s) as simplified exact numbers.

A function $f(x)$ is periodic with period $T > 0$ if $f(x + T) = f(x)$ for all x. The smallest such number T is called the least period. For example, the functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π.

(a) Let a function $g(x)$ be periodic with the least period $T = \pi$. Determine the least period of $g(x/3)$.

(b) Determine the least period of $H(x) = \sin(8x) + \cos(4x)$

(c) Determine the least periods of each of $G(x) = \sin(\cos(x))$ and $F(x) = \cos(\sin(x))$.

6. Suppose that X and Y are angles with $\tan X = \dfrac{1}{m}$ and $\tan Y = \dfrac{a}{n}$ for some positive integers a, m and n. Determine the number of positive integers $a \leq 50$ for which there are exactly 6 pairs of positive integers (m, n) with $X + Y = 45^\circ$.

(Note: The formula $\tan(X + Y) = \dfrac{\tan X + \tan Y}{1 - \tan X \tan Y}$ may be useful.)

Problem 7

The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

(A) $\frac{\pi}{2}$ **(B)** π **(C)** 2π **(D)** 4π **(E)** It's not periodic.

Problem 9

Which of the following describes the largest subset of values of y within the closed interval $[0, \pi]$ for which

$$\sin(x+y) \leq \sin(x) + \sin(y)$$

for every x between 0 and π, inclusive?

(A) $y = 0$ **(B)** $0 \leq y \leq \frac{\pi}{4}$ **(C)** $0 \leq y \leq \frac{\pi}{2}$ **(D)** $0 \leq y \leq \frac{3\pi}{4}$ **(E)** $0 \leq y \leq \pi$

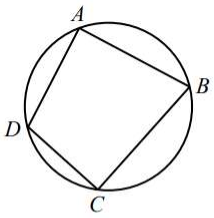
Problem 15

Let $f(x) = \sin x + 2\cos x + 3\tan x$, using radian measure for the variable x. In what interval does the smallest positive value of x for which $f(x) = 0$ lie?

(A) $(0, 1)$ **(B)** $(1, 2)$ **(C)** $(2, 3)$ **(D)** $(3, 4)$ **(E)** $(4, 5)$

Geometry:

1. A *cyclic quadrilateral* is a quadrilateral whose four vertices lie on some circle. In a cyclic quadrilateral, opposite angles add to 180°. In the diagram, $ABCD$ is a cyclic quadrilateral. Therefore, $\angle ABC + \angle ADC = 180° = \angle BAD + \angle BCD$.



(a) In Figure A below, $ABCD$ is a cyclic quadrilateral. If $\angle BAD = 88°$, what is the value of u?

(b) In Figure B, $PQRS$ and $STQR$ are cyclic quadrilaterals. If $\angle STQ = 58°$, what is the value of x and what is the value of y?

(c) In Figure C, $JKLM$ is a cyclic quadrilateral with $JK = KL$ and $JL = LM$. If $\angle KJL = 35°$, what is the value of w?

(d) In Figure D, $DEFG$ is a cyclic quadrilateral. FG is extended to H, as shown. If $\angle DEF = z°$, determine the measure of $\angle DGH$ in terms of z.

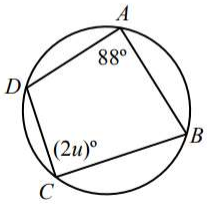


Figure A

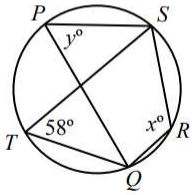


Figure B

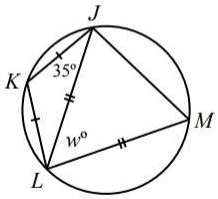


Figure C

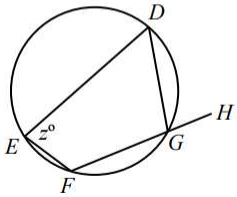


Figure D

2. If a line segment is drawn from the centre of a circle to the midpoint of a chord, it is perpendicular to that chord. For example, in Figure 1, OM is perpendicular to chord AB.

If a line segment is drawn from the centre of a circle and is perpendicular to a chord, it passes through the midpoint of that chord. For example, in Figure 2, $PR = QR$.

Figure 1

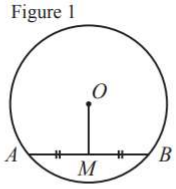
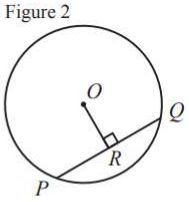
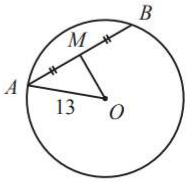


Figure 2

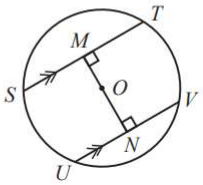


(a) In the diagram, a circle with radius 13 has a chord AB with length 10. If M is the midpoint of AB, what is the length of OM?

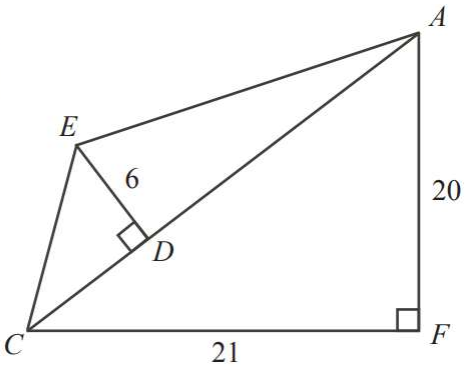


(b) In a circle with radius 25, a chord is drawn so that its perpendicular distance from the centre of the circle is 7. What is the length of this chord?

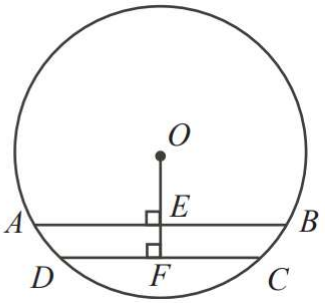
(c) In the diagram, the radius of the circle is 65. Two parallel chords ST and UV are drawn so that the perpendicular distance between the chords is 72 ($MN = 72$). If MN passes through the centre of the circle O, and ST has length 112, determine the length of UV.



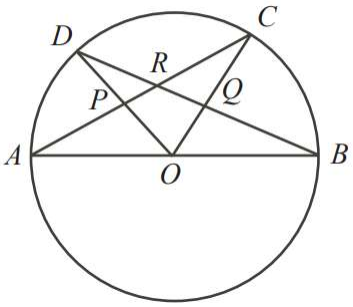
. In the diagram, $\angle AFC = 90°$, D is on AC, $\angle EDC = 90°$, $CF = 21$, $AF = 20$, and $ED = 6$. Determine the total area of quadrilateral $AFCE$.



In the diagram, the circle has centre O. OF is perpendicular to DC at F and is perpendicular to AB at E. If $AB = 8$, $DC = 6$ and $EF = 1$, determine the radius of the circle.



In the diagram, AB is a diameter of a circle with centre O. C and D are points on the circle. OD intersects AC at P, OC intersects BD at Q, and AC intersects BD at R. If $\angle BOQ = 60°$ and $\angle APO = 100°$, calculate the measure of $\angle BQO$.



In the diagram, the triangle has side lengths 6, 8 and 10. Three semi-circles are drawn using the sides of the triangle as diameters. A large circle is drawn so that it just touches each of the three semi-circles. What is the radius of the large circle?